STATICS

ANALYSIS AND DESIGN OF SYSTEMS IN EQUILIBRIUM

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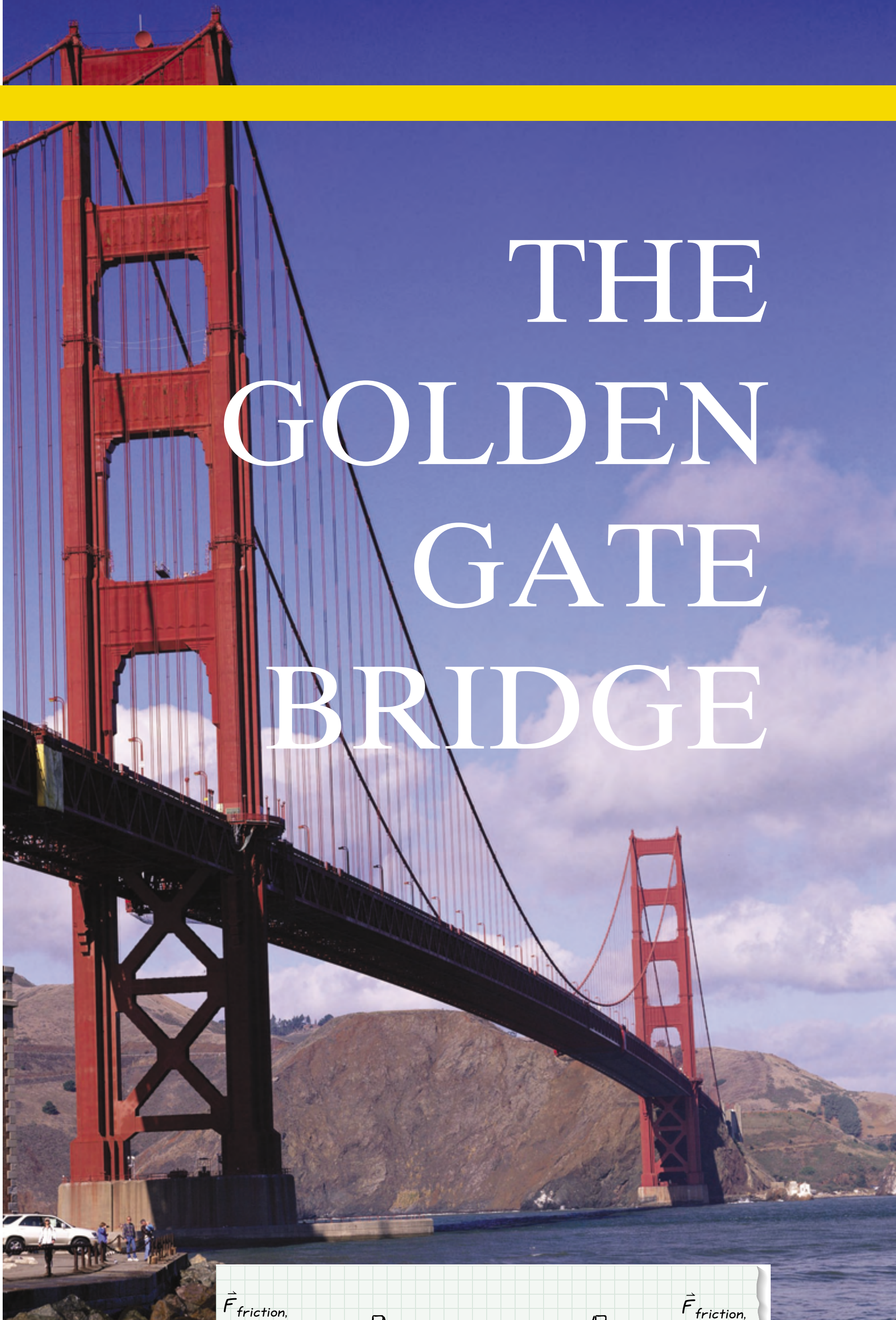
CHAPTER 3

THE GOLDEN GATE BRIDGE

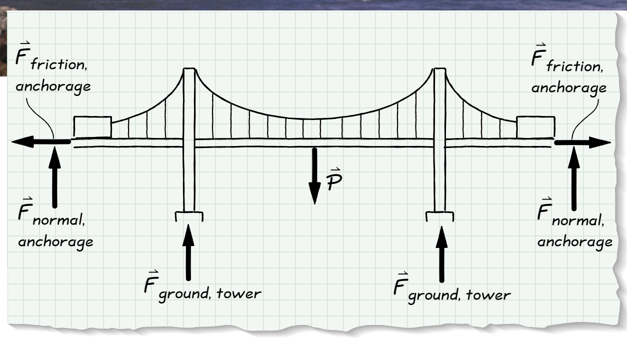


In this chapter, we use statics concepts to understand how a bridge functions and why the Golden Gate Bridge is shaped the way it is. We have chosen the Golden Gate Bridge because it is one of the most recognized and beautiful structures in the world.[1] Its graceful lines, Art Deco details, and spectacular views make it a popular stop for tourists from around the world. When it was completed in 1937, the 1280-meter suspension span was the longest in the world (now the seventh longest). Today more than 100,000 vehicles cross the bridge every day. It has been subjected to earthquakes, strong winds, and swift tides, and yet it continues to perform its function of linking the headlands on the two sides of the entrance to San Francisco Bay.[2] Engineers designing the bridge used statics concepts first to evaluate all the loads the bridge could potentially experience and then to design the members to resist these loads.

Before reading further, think about different types of bridges you have seen.

1. Sketch at least two of them and label the parts you know.
2. Identify the locations where the forces exerted on the bridge are transferred to the ground.

You may want to go to a bridge near you and study how it is built and, particularly, how it is attached to the ground. There are also many excellent pictures of bridges on the Internet. This task will provide good background for the discussion in this chapter.



[1]The Golden Gate Bridge District maintains a Web site (http://www.goldengate.org/) if you are interested in more information about this bridge.

[2]The bridge is named after the entrance to San Francisco Bay, which was named the Golden Gate by John Charles Fremont in the mid-1800s.

OBJECTIVES

On completion of this chapter, you will be able to:

- **Describe the types of forces that act on a bridge**
- **Isolate components of a suspension bridge and identify the external forces**
- **Use Newton's first law to answer questions about the transfer of loads from one bridge component to another**
- **Compare the analysis of an approximate model with a more "exact" model**

3.1 A WALK ACROSS THE BRIDGE

The main components of a suspension bridge that carry the loads from the bridge deck down to the ground are shown in **Figure 3.1**. The cars, trucks, trains, and people travel on the bridge deck, which hangs from suspenders hung from main cables that are draped over towers and attached to anchorages at each end of the bridge. The towers are embedded deep in the ground and supported by massive concrete foundations.

Imagine that you are standing in the middle of the Golden Gate Bridge. How would your weight be transferred to the ground? To get a feel for how the bridge works, let's build a model. You'll need two large paper clips, a piece of string about 3 meters long, a pencil, and six heavy books. Set aside two of the books that are about the same size (you will use them in a moment for the towers). Tie one end of the string around two of the remaining four books, tie the other end around the other two books, and place the two piles about 2 meters apart with the string lying slack between them. These piles are the two anchorages. Stand the two books you set aside on end, one about 30 cm to the right of the left anchorage and the other about 30 cm to the left of the right anchorage, as in **Figure 3.2*a***. Drape the string (the main cable) over the top of the towers, hook the two paper clips (the suspenders) onto the string, and slide the pencil (the bridge deck) into the paper clips. To represent yourself standing on the bridge, push down on the pencil.

Now modify the model by removing the anchorages and tying the string directly around the two towers (**Figure 3.2*b***). What happens when you push down on the pencil this time? Does the system collapse? Yes, the towers fall over and the bridge collapses because the towers are being pulled toward each other (inward) by the force in the main cable. In the first bridge model, the cable tied to the anchorage provides an outward force on the towers to balance the inward force. This outward

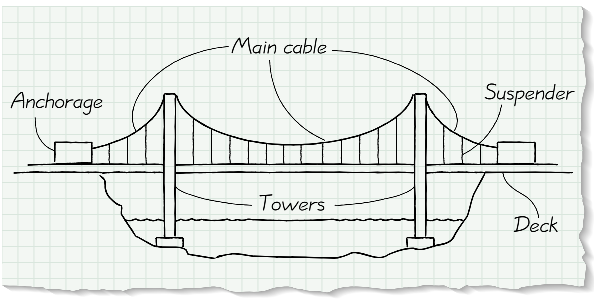
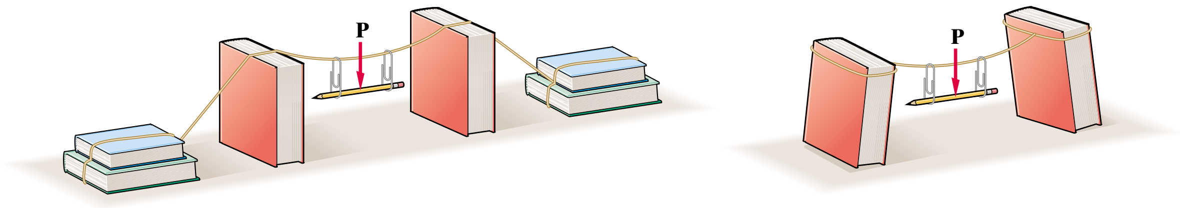


Figure 3.1 The basic components of a suspension bridge



(a) (b)

Figure 3.2 A model of a suspension bridge made from string, pencil, paper clips, books: (a) With anchorages and (b) Without anchorages

force is transferred to the table at the anchorages through friction. When the anchorages are removed and the main cable is tied directly to the towers, there is no outward force pulling on the towers to balance the inward force. Furthermore, unless we glue the upright books to the table, there is no way for the table to pull on the bottom of the towers to keep them from tipping. These two models demonstrate how all the bridge components work together to make a complete "load path" to transfer loads from bridge to ground.[3]

For a more systematic view of how the loads exerted on the bridge deck are related to the reactions exerted by the ground on each tower base ($\boldsymbol{F}_{\text{ground, tower}}$) and on each anchorage ($\boldsymbol{F}_{\text{ground, anchorage}}$ and $\boldsymbol{F}_{\text{friction, anchorage}}$), shown in **Figure 3.3**, we will trace the forces through all the bridge components. We will think in terms of the **free-body diagram** for each component of our model when we put a load on it.

1. Start with the deck. You push down on the deck (pencil), and it pulls down on the suspenders (paper clips). As a result, tension is developed in the suspenders as they pull up on the deck ($\boldsymbol{T}_{\text{suspender, deck}}$ in **Figure 3.4*a***) and down on the main cable ($\boldsymbol{T}_{\text{suspender, cable}}$ in **Figure 3.4*b***), just as in a game of tug-of-war the rope pulls on the two teams at its ends. As the suspenders pull down on the main cable (string), a tension force is created in the cable throughout its length. (The tension force is what took the slack out of the string in your model.)

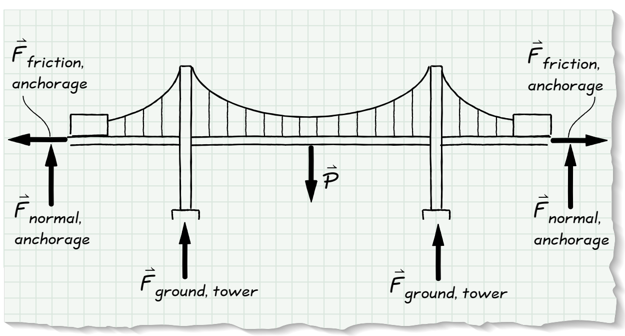


Figure 3.3 Reactions on the suspension bridge as a result of a load ***P*** exerted on the deck

[3]Exercise modified from http://www.pbs.org/wgbh/nova/bridge/meetsusp.html

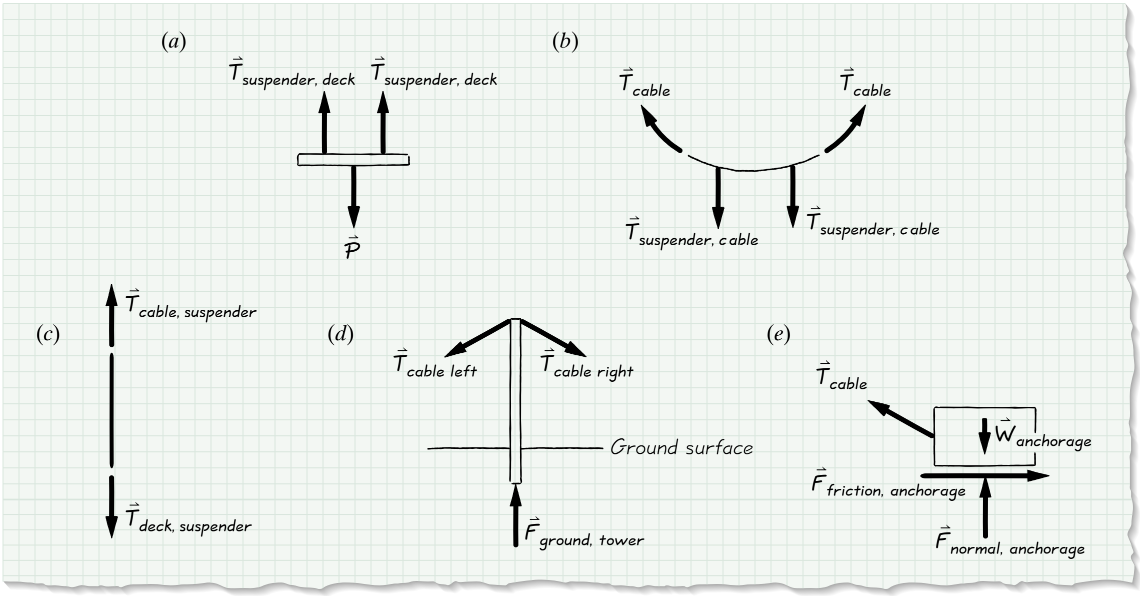


Figure 3.4 Free-body diagrams for the bridge components: (a) The suspenders pull up on the bridge deck with a force $\boldsymbol{T}_{\text{suspender, deck}}$; (b) The suspenders pull down on the main cable, causing tension $\boldsymbol{T}_{\text{cable}}$ to develop in the cable; (c) The suspender is in tension with the main cable pulling up and the deck pulling down; (d) The main cable forces are counteracted by the force $\boldsymbol{F}_{\text{ground, tower}}$ exerted by the ground on the tower; (e) The main cable force $\boldsymbol{T}_{\text{cable}}$ is counteracted by the weight of the anchorage $\boldsymbol{W}_{\text{anchorage}}$ and the forces that the ground exerts on the anchorage $\boldsymbol{F}_{\text{normal, anchorage}}$ and $\boldsymbol{F}_{\text{friction, anchorage}}$

Note that as the suspender pulls down on the main cable, the cable pulls up on the suspender with an equal and opposite force. **Figure 3.4*c*** shows a free-body diagram of a suspender. The main cable pulls up on the suspender with a force $\boldsymbol{T}_{\text{cable, suspender}}$ and the deck pulls down with a force $\boldsymbol{T}_{\text{deck, suspender}}$. Since these are the only two forces acting on the suspender, Newton's first law requires that they must be equal. For simplicity, from now on we will call these tensile forces $\boldsymbol{T}_{\text{suspender}}$.

2. Now follow the main cable to the towers. Where the main cable passes over the top of a tower (upright book), the cable is sloping away from the tower on both sides. This orientation causes the cable to pull down on the tower (**Figure 3.4*d***). To counteract this, the tower pushes up on the cable. The force of the cable pulling down on the towers is transferred to the ground through the towers and creates the reaction $\boldsymbol{F}_{\text{ground, tower}}$ shown in **Figures 3.3** and **3.4*d***. Because the tower is being pushed on at the top and bottom, it is in compression.

3. Continue tracing the main cable to the anchorage. Because the cable is embedded in the anchorage (stacked books), it pulls on the anchorage with a large force ($\boldsymbol{T}_{\text{cable}}$ in **Figure 3.4*e***). Though $\boldsymbol{T}_{\text{cable}}$ is pulling upward, the anchorage is kept from lifting off the ground by its enormous weight ($\boldsymbol{W}_{\text{anchorage}}$). The ground exerts both a normal contact force ($\boldsymbol{F}_{\text{normal, anchorage}}$) and a friction force ($\boldsymbol{F}_{\text{friction, anchorage}}$) on the anchorage, and the anchorage is kept from sliding by the friction force. As we will

discover in Chapter 6, the maximum size of the friction force is limited by the weight of the anchorage. In order to develop an adequate friction force, the anchorage must be very heavy. (Experiment with this by using lightweight books for the anchorages in your model. Do lightweight anchorages slide across the table?) The estimate of the magnitude of the cable force exerted on the anchorage is used in designing the size of the anchorage block.

Thinking about the role each structural component plays in transferring the load on the bridge deck clarifies why your model collapsed when you removed the anchorages. The bridge would collapse if the main cables were not securely anchored into the ground at each end. Even though the main cables of the Golden Gate Bridge are very slender (0.92 m diameter), they are able to transfer thousands of kiloNewtons of tensile force to the ground. To prevent uplift and sliding, each anchorage contains more than 20,000 m^3 of concrete and weighs more than 530,000 kN.

We just traveled through the bridge's **load path**, which is the route of the loads as they are transferred from one structural member to another. In studying the load path of any structure, think of the structure as a series of interconnected pipes and imagine pouring water into one end and watching the water exit at the other end. In the case of a suspension bridge, you pour water into the deck, and it flows from deck to suspenders to cables to towers and anchorages and then to the ground, where it exits the "pipe."

Summary

In this section the key ideas are:

1. A simple physical model can be used to gain an understanding of a complex structure.
2. Forces acting on the bridge are transferred from one component to another and then to the ground. The load path is the route of the loads as they are transferred from one component to another.
3. The cables and suspenders on the bridge are in tension. The bridge towers are in compression. The anchorages are kept from sliding through friction forces.

3.2 HOW HEAVY SHOULD THE ANCHORAGES BE?

Now that we have laid out in a general manner the forces acting on the components of the Golden Gate Bridge and how those forces are transferred to the ground, we will answer the same question Joseph B. Strauss and his team of engineers had to answer when they designed the bridge—how heavy should the anchorages be?

As is common in engineering analysis, we will make several assumptions to create a simplified analytical model. This will allow us to develop some equations that provide reasonable estimates of the forces acting on the components. Later in the book, we will use more complex

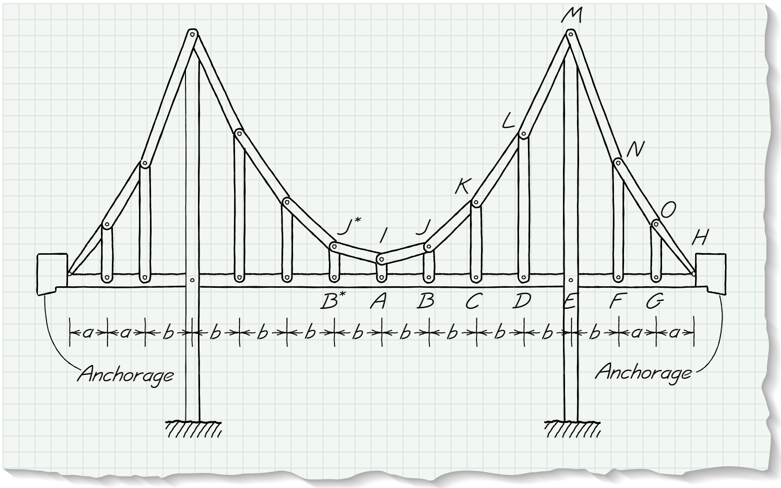


Figure 3.5 Approximate representation of the Golden Gate Bridge using pinned links to model the main cables and suspenders

assumptions and equations to perform a more exact analysis. We can then compare our approximate analysis with the more exact analysis to investigate how our simplifying assumptions have affected our results.

In order to answer the question about the weight of the anchorages, we must work our way through the load path and answer three interrelated subquestions:

1. How large is the force pulling on each of the suspenders?
2. What is the tension force in each main cable?
3. What forces on the anchorage would cause uplift or sliding?

What Assumptions Are We Making?

First, we replace each main cable by a series of links that mimic the geometry of the Golden Gate Bridge and are connected to one another with pins (**Figure 3.5** and **Table 3.1**). This will allow us to complete an approxi-

Table 3.1 Geometry of "Main Cable" in Our Approximate Model

Pin	Height above Bridge Deck (meters)	Distance from Center of Bridge (meters)
I	10.0	0
J and J*	18.9	160
K	45.8	320
L	90.4	480
M	153.0	640
N	71.4	800
O	32.8	891.5
H	0	983

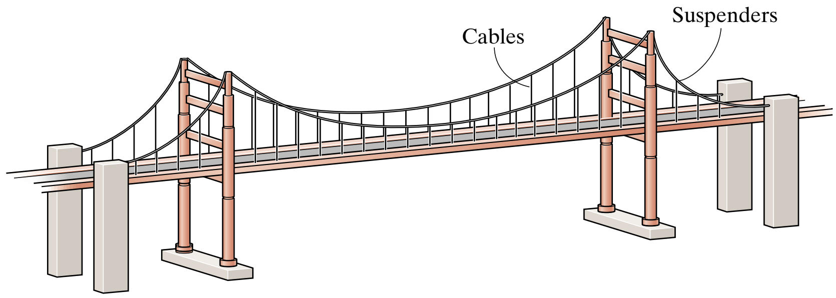


Figure 3.6 A suspension bridge has two main cables, each taking one-half of the load

mate analysis of the main cable using simple applications of Newton's first and third laws. Second, we assume that the only loads acting on the bridge are **gravity loads** (vertical loads exerted by people, vehicles, and the weight of the bridge). This allows us to ignore such horizontal loads as winds or earthquakes. Third, we assume that the weight of the main cable is much less than the weight of the bridge deck and vehicles and therefore can be ignored in our analysis. This assumption allows us to assume that the main cable is in the shape of a parabola. (For the Golden Gate Bridge, the combined weight of the deck and vehicles is more than seven times the weight of the main cables, so ignoring the weight of the cables is a reasonable assumption for a preliminary analysis.)

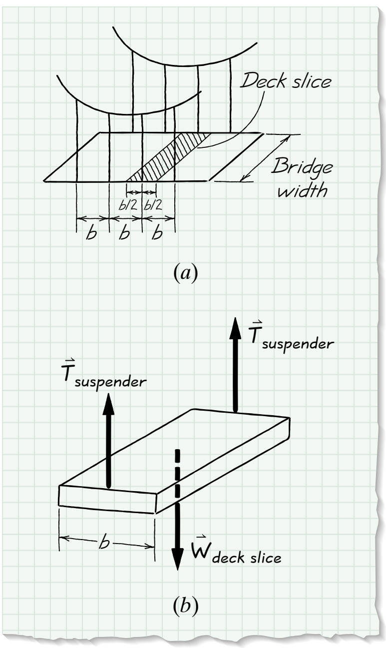


Figure 3.7 (a) A slice of bridge deck of length b is used for analysis; (b) The weight of the slice of bridge deck, pulling down, is balanced by the force exerted by the suspenders, pulling up

1. How Large Is the Force Pulling on Each of the Suspenders? When we look at the bridge from the orientation shown in **Figure 3.6**, we are reminded that there are two main cables, each attached to the deck with suspenders. When the total load is distributed uniformly across the width of the bridge, each main cable supports one half the load.

To calculate the force exerted by the deck on one suspender, we start by calling the weight per unit length of deck $\boldsymbol{w}$. We assume that all the suspenders on each side are evenly spaced along the length of the deck and that the distance between any two suspenders on the same side of the bridge is b. We then slice a length of deck out of our model (**Figure 3.7*a***), making our first cut halfway between two suspenders and our second cut a distance b away from the first cut. Thus the length of the deck slice is b, and there is one suspender attached on each side at the midpoint of length b (**Figure 3.7*b***). Applying Newton's first law, we can say that because the bridge is not moving, the sum of the forces in the vertical direction will be zero. This means that the force exerted by the suspenders pulling up ($2\boldsymbol{T}_{\text{suspender}}$) must equal the weight of the deck slice pulling down ($\boldsymbol{W}_{\text{deck slice}}$):

$$\| 2\boldsymbol{T}_{\text{suspender}} \| \; (\uparrow) - \| \boldsymbol{W}_{\text{deck slice}} \| \; (\downarrow) = 0$$
$$\| 2\boldsymbol{T}_{\text{suspender}} \| - \| \boldsymbol{w} \| \, b = 0$$
$$\| \boldsymbol{T}_{\text{suspender}} \| = \frac{\| \boldsymbol{w} \| \, b}{2} \tag{3.1}$$

We can now use this relationship to calculate the force on each suspender in our model.

Answer to Question 1

For our simplified model, the suspenders are 160 m apart and the deck weighs 330 kN/m (**Table 3.2**). The total weight of the slice we are analyzing is 52,800 kN = (330 kN/m)(160 m). Then the force of each suspender pulling up on the bridge deck is 26,400 kN = (330 kN/m)(160 m/2). We write our final answer in meganewtons (see **Table 1.2**): $\| \boldsymbol{T}_{\text{suspender}} \| = 26.4$ MN.

Table 3.2 Properties of the Golden Gate Bridge[4]

Structural Property	Quantity
Length of main span (distance between towers)	1280 m
Length of one side span	343 m
Width of bridge	27 m
Height of each tower above road deck	152 m
Height of each tower	227 m
Maximum sag in main cable	144 m
Weight of cable per one horizontal meter	48.7 kN/m
Diameter of one main cable with wrapping	0.92 m
Number of wires in each cable	27,572
Hanger spacing	15.2 m
Weight per unit length of bridge deck	330 kN/m
Weight of one tower	196,000 kN
Weight of one anchorage	530,000 kN

[4]Data from www.goldengatebridge.org/research/factsGGBDesign.html and Abdel-Ghaffar and Scanlan (1985).

2. What Is the Tension Force in Each Main Cable? In our model, we are representing each of the main cables by a series of links attached by pins. **Figure 3.8** shows the forces acting on the pin at the bridge center, the pin labeled I in Figure 3.5. We use two-letter subscripts to identify the link forces acting on any pin: the first letter indicates the pin we are currently evaluating, and the second letter indicates the other pin the link is attached to. For example, $\boldsymbol{F}_{IJ}$ symbolizes the force that link IJ exerts on pin I and $\boldsymbol{F}_{JI}$ symbolizes the force that link IJ exerts on pin J. A free-body diagram of link IJ will show that $\| \boldsymbol{F}_{IJ} \| = \| \boldsymbol{F}_{JI} \|$. In our analysis here of pin I, therefore, the two link forces are $\boldsymbol{F}_{IJ^*}$ and $\boldsymbol{F}_{IJ}$.

The suspender force $\boldsymbol{T}_{\text{suspender}}$ pulls down on the pin, and the forces $\boldsymbol{F}_{IJ}$ and $\boldsymbol{F}_{IJ^*}$ in the links IJ and IJ^* each pull away from the pin along the long axes of the links. Because the pin's state of motion is not changing, Newton's first law requires that the sum of the forces in the horizontal direction as well as the sum of the forces in the vertical direction be zero (**Figure 3.9**). Looking first at the horizontal forces:

$$\| \boldsymbol{F}_{IJ\,\text{horizontal}} \| \; (\rightarrow) - \| \boldsymbol{F}_{IJ^*\,\text{horizontal}} \| \; (\leftarrow) = 0$$

$$\| \boldsymbol{F}_{IJ} \| \; \cos\,\alpha - \| \boldsymbol{F}_{IJ^*} \| \; \cos\,\alpha = 0$$

$$\| \boldsymbol{F}_{IJ^*} \| = \| \boldsymbol{F}_{IJ} \| \qquad (3.2\text{A})$$

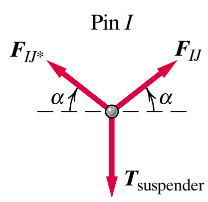


Figure 3.8 Free-body diagram of pin I

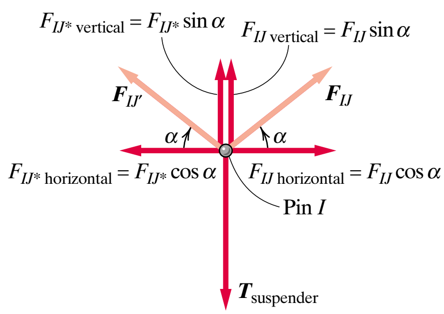


Figure 3.9 Force balance: Equilibrium means that the forces pulling to the left equal the forces pulling to the right on pin I. Similarly, the forces pulling down equal the forces pulling up

Now looking at the vertical forces:

$$\| \boldsymbol{F}_{IJ^*\,\text{vertical}} \| \ (\uparrow) + \| \boldsymbol{F}_{IJ\,\text{vertical}} \| \ (\uparrow) - \| \boldsymbol{T}_{\text{suspender}} \| \ (\downarrow) = 0$$

$$\| \boldsymbol{F}_{IJ^*} \| \ \sin\alpha + \| \boldsymbol{F}_{IJ} \| \ \sin\alpha - \| \boldsymbol{T}_{\text{suspender}} \| \ = 0 \qquad (3.2B)$$

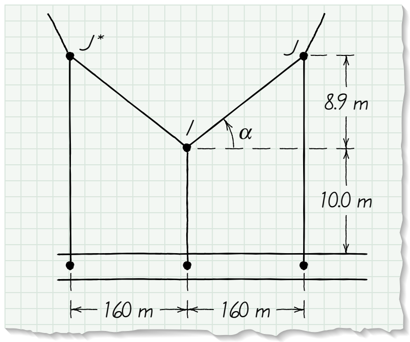


Figure 3.10 Blow-up of a segment at the center of the bridge

Substituting from (3.2A) into (3.2B) and rearranging gives

$$\| \boldsymbol{F}_{IJ} \| = \frac{\| \boldsymbol{T}_{\text{suspender}} \|}{2\sin\alpha} \qquad (3.3)$$

The angle α can be determined from the geometry of the bridge shown in **Figure 3.5**.[5] **Figure 3.10** shows a blow-up of a segment cut out of the center of the bridge. From the dimensions shown in the figure, $\cos\alpha = 160/160.25 = 0.998$ and $\sin\alpha = 8.9/160.25 = 0.0556$. From (3.3) and our known value $\| \boldsymbol{T}_{\text{suspender}} \| = 26.4$ MN, we see that $\| \boldsymbol{F}_{IJ} \| = \| \boldsymbol{F}_{IJ^*} \| = 237$ MN. The result of this calculation indicates that the force in the main cable is quite large—a force of 237 MN is equivalent to the weight of about 18,000 automobiles.

The next step is to draw a free-body diagram of the pin at J, as shown in **Figure 3.11**. Once again, requiring the sum of the forces in the horizontal direction to be zero gives

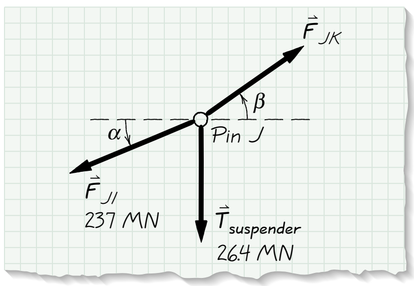


Figure 3.11 Free-body diagram of pin J

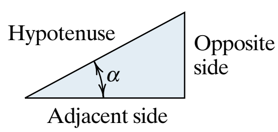
$$\| \boldsymbol{F}_{JK\,\text{horizontal}} \| \ (\rightarrow) - \| \boldsymbol{F}_{JI\,\text{horizontal}} \| \ (\leftarrow) = 0$$

$$\| \boldsymbol{F}_{JK} \| \ \cos\beta - \| \boldsymbol{F}_{JI} \| \ \cos\alpha = 0$$

We determine β from **Table 3.1**, which shows that the vertical distance from J to K is 45.8 m − 18.9 m = 26.9 m. This distance is the length of the side opposite β in the right triangle suggested in **Figure 3.11**. **Table 3.1** also shows that the side adjacent to β is 320 m − 160 m = 160 m. Using these values in the hypotenuse formula from footnote 5 gives 162.25 m for the hypotenuse length in the right triangle suggested in **Figure 3.11**. Therefore $\cos\beta$ = adjacent side/hypotenuse = 160/162.25 = 0.986. This gives

$$\| \boldsymbol{F}_{JK} \| = \| \boldsymbol{F}_{JI} \| \frac{\cos\alpha}{\cos\beta} = (237\ \text{MN})\frac{0.998}{0.986} = 240\ \text{MN}$$

[5]For a right triangle, the lengths of the two sides and the hypotenuse are sufficient to determine angles and sines and cosines of those angles.



For the triangle shown here, "adjacent side" is the length of the side adjacent to the angle θ, "opposite side" is the length of the side opposite the angle θ, and "hypotenuse" is the length of the hypotenuse = (adjacent side^2 + opposite side^2)$^{1/2}$. Then $\sin\theta$ = opposite side/hypotenuse and $\cos\theta$ = adjacent side/hypotenuse.

Answer to Question 2

Figure 3.12 compares our simplified analytic model (red lines) with the results of a more exact analysis (green curves) as presented in Chapter 10. This figure shows that our simplified analysis provides a good estimate of the force exerted on the main cable. Near the center of the bridge, the results vary by less than 0.1%, and near either tower the variation is no more than 3%, Variations occur because the links, being a series of straight lines, cannot exactly duplicate the geometry of the parabolic cable. If we were to modify our approximate model by making the links shorter and the suspenders closer together, we could more closely approximate the actual bridge geometry and would converge to the parabolic cable solution in **Figure 3.12**.

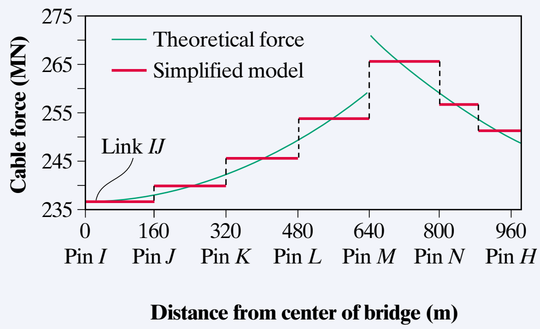


Figure 3.12 Forces calculated from the simplified analytical model of the main cable are a good approximation of the actual forces exerted on the cable

To complete the analysis, we repeat the same type of calculation for each pin as we move across the bridge. The next pin is pin K. Note that as we move along the center span of the bridge from the center toward the towers, the force on each successive link increases, with the largest force being on link LM. On the side span, we find the force decreasing as we move from the tower toward the anchorage.

Why do you think there is a discontinuity in the theoretical curve at the tower (pin M)? This discontinuity occurs because of the change in the orientation of the cable as it is draped over the top of the tower. The horizontal component (T_h) of the cable force remains constant throughout the length of the cable. The magnitude of the cable force at any location is $T = T_h/\cos\theta$, where θ is the angle between the cable and the horizontal. At the tower, the angle changes abruptly and consequently so does the cable force.

3. What Forces on the Anchorage Would Cause Uplift or Sliding? The main cable is pulling on the anchorage at the right end of the bridge with a very large force directed upward and to the left (**Figure 3.4*e*** and **Figure 3.13**). In our model this force is represented by $\boldsymbol{F}_{HO}$, the force of link HO pulling on the anchorage at H. The anchorage is kept from sliding to the left by the large friction force $\boldsymbol{F}_{\text{friction, anchorage}}$ developed between it and the ground. It is kept from lifting off the ground by its heavy weight. In designing the Golden Gate Bridge anchorages, engineers had to make each anchorage heavy enough to prevent either sliding or uplift.

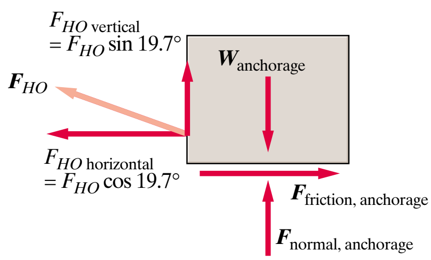


Figure 3.13 Friction forces and the weight of the anchorage counteract the pull of the main cable

What minimum weight of the anchorage will prevent it from lifting? Newton's first law tells us that if the anchorage is stationary, the sum of the forces in the vertical and horizontal directions must be zero. Looking at **Figure 3.13**, this means that

$$\|\boldsymbol{F}_{HO\text{ vertical}}\|(\uparrow) + \|\boldsymbol{F}_{\text{normal, anchorage}}\|(\uparrow) - \boldsymbol{W}_{\text{anchorage}}(\downarrow) = 0 \quad (3.4)$$

$$\|-\boldsymbol{F}_{HO\text{ horizontal}}\|(\leftarrow) + \|\boldsymbol{F}_{\text{friction, anchorage}}\|(\rightarrow) = 0 \quad (3.5)$$

If the anchorage were to lift off the ground, the normal force shown in **Figure 3.13** would be zero. We can calculate the weight of the anchorage when the normal force is zero by equating the forces in the vertical direction to zero and eliminating the normal force from (3.4):

$$\begin{aligned} \| \boldsymbol{F}_{HO\ \text{vertical}} \| \,(\uparrow) - \| \boldsymbol{W}_{\text{anchorage at uplift}} \| \,(\downarrow) &= 0 \\ \| \boldsymbol{W}_{\text{anchorage at uplift}} \| &= \| \boldsymbol{F}_{HO\ \text{vertical}} \| \\ \| \boldsymbol{W}_{\text{anchorage at uplift}} \| &= (251.2\ \text{MN})(\sin 19.7°) = 84.7\ \text{MN} \end{aligned} \tag{3.6}$$

If the anchorage weighs less than 84.7 MN, it will lift off the ground, and if it weighs more it will not.

The friction force required to prevent sliding can be determined from (3.5):

$$\begin{aligned} \| \boldsymbol{F}_{\text{friction, anchorage}} \| &= \| \boldsymbol{F}_{HO\ \text{horizontal}} \| \\ \| \boldsymbol{F}_{\text{friction, anchorage}} \| &= (251.2\ \text{MN})(\cos 19.7°) = 236.5\ \text{MN} \end{aligned} \tag{3.7}$$

The next question we want to ask is how heavy the anchorage must be to develop this large friction force. As presented in Chapter 6, the maximum friction force ($\boldsymbol{F}_{\text{friction max}}$) that can be produced between the ground and the anchorage depends on normal force between the anchorage and the ground and the roughness of contact between the two surfaces, reflected in the **coefficient of friction**, μ_{static}. For rough materials such as rock and concrete, μ_{static} could be in the range from 0.5 to 0.7. For our example we shall use $\mu_{\text{static}} = 0.6$. The relationship is expressed mathematically as

$$\boldsymbol{F}_{\text{friction max}} = \mu_{\text{static}} \boldsymbol{F}_{\text{normal, anchorage}} \tag{3.8}$$

In (3.7) we determined that the friction force needed to prevent sliding is 236.5 MN, which must be developed by the roughness between the anchorage and the ground as expressed by (3.8). Therefore

$$\begin{aligned} \| \boldsymbol{F}_{\substack{\text{friction,} \\ \text{anchorage}}} \| = \| \boldsymbol{F}_{\text{friction max}} \| = \mu_{\text{static}} \| \boldsymbol{F}_{\substack{\text{normal,} \\ \text{anchorage}}} \| &= 236.5\ \text{MN} \\ \| \boldsymbol{F}_{\text{normal, anchorage}} \| = \frac{236.5\ \text{MN}}{0.6} &= 394\ \text{MN} \end{aligned}$$

Finally, the weight of anchorage required to produce a normal force of 394 MN is found from (3.4):

$$\begin{aligned} \| \boldsymbol{W}_{\text{anchorage required}} \| &= \boldsymbol{F}_{\text{normal, anchorage}} \| + \| \boldsymbol{F}_{HO\ \text{vertical}} \| \\ &= 394\ \text{MN} + 84.7\ \text{MN} = 479\ \text{MN} \end{aligned} \tag{3.9}$$

Answer to Question 3

This tells us that the anchorage must weigh more than 84.7 MN to prevent uplift and more than 479 MN to prevent sliding. In fact, on the Golden Gate Bridge each anchorage weighs about 530 MN, which satisfies both conditions.

3.3 ADDING MORE REALITY

Not all bridges are suspension bridges. Engineers use different design solutions after considering many issues, such as distance to be spanned, types of loads to be carried, strength of the rock available for the foundation, type of material to be used, aesthetics, and cost. Suspension bridges are typically used for spanning large distances, on the order of 600 to 2000 meters. For shorter distances, designers might use beam, arch, or truss bridges. Beam bridges, typically seen as freeway overpasses, are inexpensive to build and efficient for spanning distances of 75 meters or less. Arch bridges, developed by the ancient Romans, are useful for spanning distances from 100 to 400 meters. Truss bridges have the advantage of being lightweight and can be built up from a series of short members.

If you study the Golden Gate Bridge, you will see that it is made up of several types of bridges. For example, the south approach consists of a steel arch, five truss spans, and a series of steel beam bridges. As we shall learn in later chapters, each bridge type has a different mechanism for transferring loads to the ground.

Up to this point we have assumed that the Golden Gate Bridge is not moving and that only gravity forces act on it. In fact, the bridge is moving all the time and is subjected to a number of dynamic loads, including earthquakes, wind loads, vehicle loads, and the action of strong tidal currents. The currents and the wind impart sideways loads on the towers, causing them to sway approximately 0.3 m from side to side. Vehicular traffic is another source of bridge movement, and as you stand on the bridge sidewalk, you can feel the vibrations of the deck as the cars and trucks drive by.

In extreme cases, wind loads can cause a bridge deck to oscillate and twist wildly, possibly leading to a collapse, as was the case in 1940 on the Tacoma Narrows Bridge. The deck acts like an airfoil as the wind passes by and causes the deck to lift and fall. As the deck goes up and down, changes in the geometry of the main cables cause the towers to sway shoreward and channelward as much as 0.5 m. Thermal expansion and contraction of the main cables also causes the deck to move. Design calculations indicate that at its center, the deck of the Golden Gate Bridge can deflect downward 3.3 m and upward 1.8 m as a result of temperature and other loading.

Because the Golden Gate Bridge is not far from the San Andreas and Hayward faults, it is periodically subjected to earthquakes. During an earthquake, the ground accelerates vertically and horizontally, causing **inertial forces** to act on the bridge. The inertia of a structure causes it to resist any sudden movement of its base, so that the upper parts of the structure deform relative to the base (**Figure 3.14**). A unique feature of inertial forces caused by earthquakes is that they are proportional to the weight of the structure—the heavier the structure, the larger the forces. The motion of the bridge during an earthquake is very complex, consisting of horizontal and vertical vibrations as well a twisting of the deck and towers. Calculation of bridge deflections and the resulting forces requires a **dynamic analysis**. Although a static analysis can provide preliminary

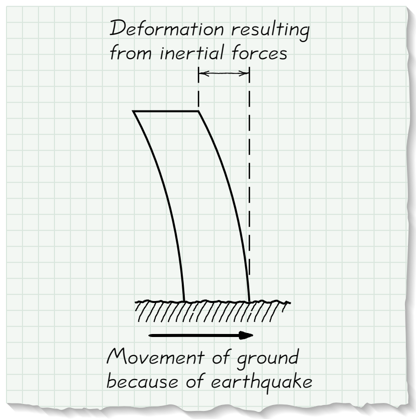


Figure 3.14 Deformation of a structure subjected to earthquake inertial forces

estimates of the earthquake and wind forces acting on each bridge component, a dynamic analysis will provide more accurate results.

3.4 JUST THE FACTS

In this chapter we examined the question of how heavy the anchorages should be for the Golden Gate Bridge. By analyzing the loads acting on a suspension bridge as they are transferred from the bridge deck to the suspenders and then through the main cables to the anchorages, we were able to find the forces of the cables pulling on the anchorages. We used a simplified analytical model to calculate an approximate solution to the forces in the bridge's main cable. We then compared our approximate analysis with a more exact solution. We examined how heavy the anchorages must be to prevent both uplift and sliding. The analysis involved making assumptions, creating free-body diagrams, and then applying Newton's first law.

3.5 REFERENCES

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SYSTEMS ANALYSIS (SA) EXERCISES

SA3.1 Exploring a Suspension Bridge

1. Reconstruct the model of a suspension bridge in Section 3.1 (**Figure 3.2*a***). Push down on the pencil and feel how much force the system can resist before the anchorages start to slide.

   Now remove one of the books from each of the anchorages. Push on the pencil again. Can the system resist more or less force than before? How does the system fail?

   Try making other alterations to the model to examine the effect on the system capacity (i.e., the force it can support) and the failure mechanism. Examples of alterations you can implement include:

   (a) Adding a book to each of the anchorages so there are three books for each

   (b) Inserting a shiny (slippery) piece of paper between the table surface and the books that serve as anchorages

   (c) Inserting a rough piece of cloth or carpet between the table surface and the books that serve as anchorages

   (d) Shortening the string so that it is tight across the books that serve as the towers

   (e) Moving the towers very close to the anchorages or very close together

2. Assuming that the geometry of the Golden Gate Bridge remains unchanged, double the weight per unit length of the bridge deck to 660 kN/m and calculate the force $\boldsymbol{F}_{IJ}$ acting on member *IJ* (**Figures 3.5** and **3.8**).

   (a) How much does $\boldsymbol{F}_{IJ}$ change?

   (b) How much will doubling the weight per unit length of the bridge deck change the force $\boldsymbol{F}_{HO}$ pulling on the anchorage? Explain your answer.

3. If $\boldsymbol{F}_{HO}$ is doubled (**Figure 3.12**), by how much will the required weight of the anchorage increase? Explain your answer.

SA3.2 Exploring a Beam Bridge

Whereas suspension bridges are efficient for spanning distances of about 600 m to 2000 m, beam bridges are often used to span short distances. A common example of a beam bridge is a freeway overpass. To model a beam bridge, you need three books. Place two books on end about 20 cm apart to serve as the piers and lay the third book across the two to create the bridge deck as shown in **Figure SA3.2.1**.

Load the bridge in two ways:

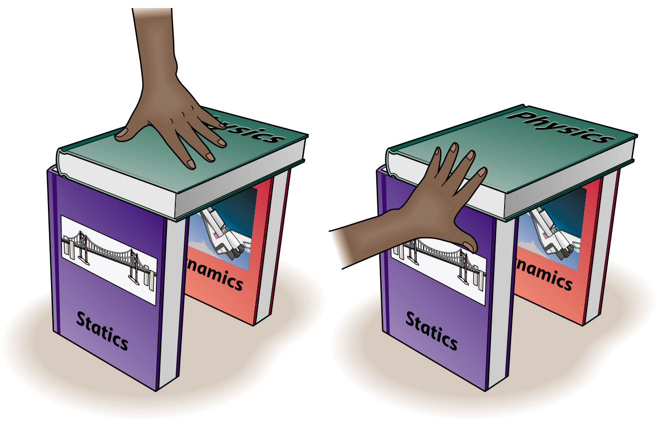
1. Push straight down on top of the deck with your hand.
2. Push horizontally on the deck with your hand.

For each of these loading cases:

(a) Draw a free-body diagram for each component of the bridge to trace the load path as $\boldsymbol{F}_{\text{hand}}$ is transferred to the ground. State any assumptions you are making in drawing the diagrams.

(b) Explain why the bridge in part (b) of the figure fell over and how you might alter the design so that it wouldn't.

(c) Now replace the book that is modeling the bridge deck with a piece of cardboard or thick paper. Push straight down on the deck with your hand. Describe the behavior of the deck.



(*a*) (*b*)

SA3.2.1 Beam bridge modeled with books: (a) Loaded with a vertical force and (b) Loaded with a horizontal force

SA3.3 Exploring an Arch Bridge

An arch bridge is made up of a bridge deck supported by an arch that is connected at both ends to supports called abutments. The load on the bridge is transferred along the curve of the arch to the abutments. The arch bridge can span larger distances than a beam bridge (100 to 400 meters). To understand the load transfer in an arch bridge and the function of the abutments, you can build a model.

- You need a one-pint (or larger) container like the type used to package cottage cheese, sour cream, or delicatessen food.
- Cut the container in half along its diameter so that it makes two semicircular pieces.
- To complete the arch, cut off the bottom of the container and the stiffening ring (or lip) at the top (**Figure SA3.3.1*a***).

Load the arch by pressing down on the center as shown in **Figure SA3.3.1*b***.

(a) Is the arch in tension or compression? What happens to the ends of the arch?

To prevent the arch from collapsing, you must add abutments. To model your abutments, have a friend place her or his hands at the intersections of the arch and the ground as shown in **Figure SA3.3.1*c***. Again load the arch by pressing down on the center as shown in **Figure SA3.3.1*d***.

(b) How do the abutments affect how the ends of the arch move?

(c) Is the arch pushing or pulling on the abutments?

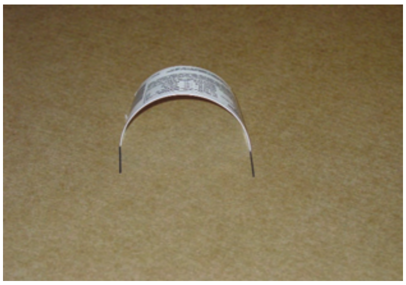
(d) What prevents the abutments from sliding?

(e) With the abutments in place, does the bridge provide more or less resistance to the push of your finger?

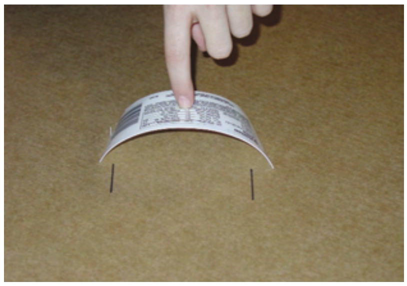
(f) Draw a free-body diagram showing all of the forces acting on one abutment.

(g) Review the analysis of the Golden Gate Bridge anchorage (**Figure 3.12** and (3.4) to (3.9)) and explain how the weight of the arch bridge abutment is important in preventing it from sliding. To test your reasoning, model the abutments using something relatively lightweight such as CD-ROM cases. When you load the bridge, do the lightweight abutments move? Now press down on the lightweight abutments and load the bridge. Do the abutments move when the extra weight is on them?

(h) Are there any forces pulling up on the abutment? To determine the required weight of an arch bridge abutment, how would the analysis differ from the analysis of the suspension bridge anchorage?



SA3.3.1*a* Arch bridge made from cottage cheese container



SA3.3.1*b* Loaded arch without abutments

SA3.3.1*c* Abutments added to arch bridge

SA3.3.1*d* Loaded arch with abutments

SA3.4 Exploring a Cable-Stayed Bridge

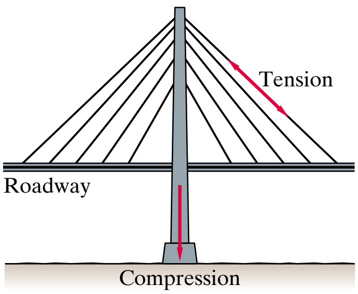
It is possible to confuse cable-stayed bridges with suspension bridges because both types use cables to hold up the bridge deck. However, the two bridge types have different mechanisms for transferring loads to the ground. In a suspension bridge, the main cables are draped over the tops of the towers and pull on the anchorages. Both the towers and the anchorages transfer the loads to the ground. In a cable-stayed bridge, the cables are attached directly to the towers and only the towers transfer the loads to the ground. As shown in **Figure SA3.4.1**, the cables, which are in tension, pull up on the deck and down on the tower. The tower, which is in compression, transfers a downward force to the ground.

To understand the load transfer in a cable-stayed bridge, we can build a model. The cables can be attached to the towers in a number of patterns, but for our model we will use a fan pattern. In this pattern all of the cables are attached to the top of the tower, and then each cable is attached to a different point along the length of the bridge deck.

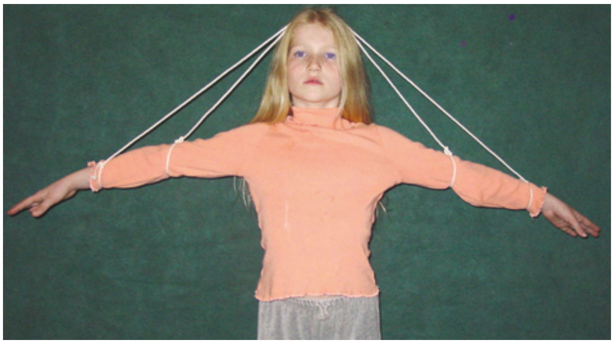
- You need two pieces of string, one about 1.5 meters long and the other 2 meters long, and a partner to help you.

Use your arms to model the bridge deck by holding both arms out horizontally to the side. You should be able to feel your muscles holding up your arms. Model the bridge tower with the trunk of your body and your head, and use the tower to support the cables that support the bridge deck. Have your partner tie the 1.5-m piece of string to each of your elbows, with the middle of the string lying on top of your head. The string acts as a stay-cable and holds your elbows up, but your hands and lower arms are hanging downward with little support. You should feel less stress on your muscles.

Have your partner tie the 2-m piece of string to each wrist, with the middle of the string lying on top of your head, making sure both strings are still taut. The bridge is now supported by two stay-cables, and your lower arms are also supported as shown in **Figure SA3.4.2**.



SA3.4.1 Load path for cable-stayed bridge



SA3.4.2 Model of cable-stayed bridge

Describe the load path of the cable-stayed bridge by answering these questions:

(a) What forces acting on the bridge deck are being transferred to the ground?

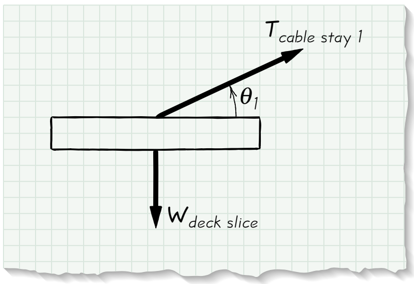
(b) Which components of the bridge are in tension?

(c) In which parts of your body do you feel a compression force? (**Figure SA3.4.1** does not show all of the compressive forces acting on the bridge.)

(d) How is the load on the top of your head transferred to the ground?

Assuming that the cables are attached to the deck at equal intervals and each carries an equal portion of the weight of bridge deck, the cables attached farther from the tower are subjected to larger tension forces than those attached closer to the tower. **Figure SA3.4.3** is an incomplete free-body diagram of a deck slice showing the force pulling on cable stay number 1 and the weight of the slice.

(e) Using the variables shown in **Figure SA3.4.3**, convince yourself that the steeper cables, which are attached closer to the tower, are subjected to smaller tension forces. For this exercise, the cable number increases as you approach the tower.



SA3.4.3 Tension force on cable stay 1 and the weight of the bridge deck slice

[6]Adapted from http://www.pbs.org/wgbh/nova/bridge/meetcable.html